Chapter

3





#### Chapter

3

 $\sim$ 

 $\sim$ 

 $\sim$ 



Chapter 3

#### **Chapter 3: Accelerated Motion**

Section 3.1: Acceleration

Section 3.2: Motion with Constant Acceleration

Section 3.3: Free Fall



## In this section you will:

- **Define** acceleration.
- **Relate** velocity and acceleration to the motion of an object.
- **Create** velocity-time graphs.

# **Changing Velocity**

- You can feel a difference between uniform and nonuniform motion.
- When you move in a nonuniform motion, you feel pushed or pulled.
- In contrast, when you are in uniform motion and your eyes are closed, you feel as though you are not moving at all.



#### **Changing Velocity**

Consider the particle-model motion diagram below showing the distance between successive positions.



#### **Changing Velocity**

There are two major indicators of the change in velocity in this form of the motion diagram. The change in the spacing of the dots and the differences in the lengths of the velocity vectors indicate the changes in velocity.



#### **Changing Velocity**

- If an object speeds up, each subsequent velocity vector is longer.
- If the object slows down, each vector is shorter than the previous one.
- Both types of motion diagrams give an idea of how an object's velocity is changing.





# **Velocity-Time Graphs**

The rate at which an object's velocity changes is called the acceleration of the object. When the velocity of an object changes at a constant rate, it has a constant acceleration.



#### **Average and Instantaneous Acceleration**

- The average acceleration of an object is the change in velocity during some measurable time interval divided by that time interval.
- Average acceleration is measured in  $m/s^2$ .
- The change in velocity at an instant of time is called instantaneous acceleration.

#### **Average and Instantaneous Acceleration**

The instantaneous acceleration of an object can be found by drawing a tangent line on the velocity-time graph at the point of time in which you are interested. The slope of this line is equal to the instantaneous acceleration.





# **Displaying Acceleration on a Motion Diagram**

- For a motion diagram to give a full picture of an object's movement, it also should contain information about acceleration. This can be done by including average acceleration vectors. These vectors will indicate how the velocity is changing.
- To determine the length and direction of an average acceleration vector, subtract two consecutive velocity vectors, as shown below.



### **Displaying Acceleration on a Motion Diagram**

- You will have:  $\Delta v = v_f v_i = v_f + (-v_i)$ .
- Then divide by the time interval, Δt. The time interval, Δt, is 1 s. This vector, (v<sub>f</sub> - v<sub>i</sub>)/1 s, shown in violet, is the average acceleration during that time interval.
- The velocities v<sub>i</sub> and v<sub>f</sub> refer to the velocities at the beginning and end of a chosen time interval.



How would you describe the sprinter's velocity and acceleration as shown on the graph?



#### **Velocity and Acceleration**



# Step 1: Analyze and Sketch the problem



From the graph, note that the sprinter's velocity starts at zero, increases rapidly for the first few seconds, and then, after reaching about 10.0 m/s, remains almost constant.





Identify the known and unknown variables.





#### **Velocity and Acceleration**



# Step 2: Solve for the Unknown



Draw a tangent to the curve at t = 1.0 s and t = 5.0 s.





Solve for acceleration at 1.0 s:





The slope of the line at 1.0 s is equal to the acceleration at that time.





Solve for acceleration at 5.0 s:





The slope of the line at 5.0 s is equal to the acceleration at that time.

$$a = \frac{\text{rise}}{\text{run}}$$
$$= \frac{10.3 \text{ m/s} - 10.0 \text{ m/s}}{10.0 \text{ s} - 0.00 \text{ s}}$$





The acceleration is not constant because it changes from 3.4 m/s<sup>2</sup> to  $0.03 \text{ m/s}^2$  at 5.0 s.

The acceleration is in the direction chosen to be positive because both values are positive.

#### **Velocity and Acceleration**



# Step 3: Evaluate the Answer



Are the units correct?

Acceleration is measured in  $m/s^2$ .



The steps covered were:

- **Step 1:** Analyze and Sketch the Problem
- **Step 2** Solve for the Unknown
  - Draw a tangent to the curve at t = 1.0 s and t = 5.0 s.
  - Solve for acceleration at 1.0 s.
  - Solve for acceleration at 5.0 s.
- **Step 3:** Evaluate the Answer

- These four motion diagrams represent the four different possible ways to move along a straight line with constant acceleration.
- The first motion diagram shows an object moving in the positive direction and speeding up.
- The second motion diagram shows the object moving in the positive direction and slowing down.



- The third shows the object speeding up in the negative direction.
- The fourth shows the object slowing down as it moves in the negative direction.



- In the first and third situations when the object is speeding up, the velocity and acceleration vectors point in the same direction in each case.
- In the other two situations in which the acceleration vector is in the opposite direction from the velocity vectors, the object is slowing down.



- In other words, when the object's acceleration is in the same direction as its velocity, the object's speed increases. When they are in opposite directions, the speed decreases.
- Both the direction of an object's velocity and its direction of acceleration are needed to determine whether it is speeding up or slowing down.



- An object has a positive acceleration when the acceleration vector points in the positive direction and a negative acceleration, when the acceleration vector points in the negative direction.
- The sign of acceleration does not indicate whether the object is speeding up or slowing down.



#### **Determining Acceleration from a v-t Graph**

- Velocity and acceleration information also is contained in velocity-time graphs.
- Graphs A, B, C, D, and E, as shown on the right, represent the motions of five different runners.
- Assume that the positive direction has been chosen to be east.



#### Determining Acceleration from a v-t Graph

- The slopes of Graphs A and E are zero. Thus, the accelerations are zero. Both Graphs A and E show motion at a constant velocity—Graph A to the east and Graph E to the west.
- Graph B shows motion with a positive velocity. The slope of this graph indicates a constant, positive acceleration.



#### Determining Acceleration from a v-t Graph

- Graph C has a negative slope, showing motion that begins with a positive velocity, slows down, and then stops. This means that the acceleration and velocity are in opposite directions.
- The point at which Graphs C and B cross shows that the runners' velocities are equal at that point. It does not, however, give any information about the runners' positions.


#### **Determining Acceleration from a** *v***-***t* **Graph**

Graph D indicates movement that starts out toward the west, slows down, and for an instant gets to zero velocity, and then moves east with increasing speed.



#### **Determining Acceleration from a v-t Graph**

The slope of Graph D is positive. Because the velocity and acceleration are in opposite directions, the speed decreases and equals zero at the time the graph crosses the axis. After that time, the velocity and acceleration are in the same direction and the speed increases.



# **Determining Acceleration from a** *v***-***t* **Graph**

The following equation expresses average acceleration as the slope of the velocity-time graph.



Average acceleration is equal to the change in velocity, divided by the time it takes to make that change.

# **Question 1**

Which of the following statements correctly define acceleration?

- A. Acceleration is the rate of change of displacement of an object.
- B. Acceleration is the rate of change of velocity of an object.
- C. Acceleration is the amount of distance covered in unit time.
- D. Acceleration is the rate of change of speed of an object.

#### Answer 1

Answer: B

**Reason:** The rate at which an object's velocity changes is called acceleration of the object.

# **Question 2**

What happens when the velocity vector and the acceleration vector of an object in motion are in same direction?

- A. The acceleration of the object increases.
- B. The speed of the object increases.
- C. The object comes to rest.
- D. The speed of the object decreases.

Answer 2

Answer: B

**Reason:** When the velocity vector and the acceleration vector of an object in motion are in same direction, the speed of the object increases.

# **Question 3**

On the basis of the velocity-time graph of a car moving up a hill, as shown on the right, determine the average acceleration of the car?



A. 0.5 m/s<sup>2</sup>

B. -0.5 m/s<sup>2</sup>

C. 2 m/s<sup>2</sup>

D. -2 m/s<sup>2</sup>

Answer 3

Answer: B

**Reason:** Average acceleration of an object is the slope of the velocity-time graph.



# In this section you will:

- Interpret position-time graphs for motion with constant acceleration.
- Determine mathematical relationships among position, velocity, acceleration, and time.
- Apply graphical and mathematical relationships to solve problems related to constant acceleration.

Section 3.2 Velocity with Average Acceleration

- If an object's average acceleration during a time interval is known, then it can be used to determine how much the velocity changed during that time.
- The definition of average acceleration:



# Velocity with Average Acceleration

The equation for final velocity with average acceleration can be written as follows:

$$v_{\rm f} - v_{\rm i} = \bar{a}\Delta t$$

The final velocity is equal to the initial velocity plus the product of the average acceleration and time interval.



#### **Velocity with Average Acceleration**

- In cases in which the acceleration is constant, the average acceleration, *ā*, is the same as the instantaneous acceleration, *a*. The equation for final velocity can be rearranged to find the time at which an object with constant acceleration has a given velocity.
- It also can be used to calculate the initial velocity of an object when both the velocity and the time at which it occurred are given.

- The position data at different time intervals for a car with constant acceleration are shown in the table.
- The data from the table are graphed as shown on the next slide.

Position-Time Data for a Car			
Time (s)	Position (m)		
0.00	0.00		
1.00	2.50		
2.00	10.0		
3.00	22.5		
4.00	40.0		
5.00	62.5		

- The graph shows that the car's motion is not uniform: the displacements for equal time intervals on the graph get larger and larger.
- The slope of a position-time graph of a car moving with a constant acceleration gets steeper as time goes on.





- The slopes from the position time graph can be used to create a velocity-time graph as shown on the right.
- Note that the slopes shown in the position-time graph are the same as the velocities graphed in velocity-time graph.



- A unique position-time graph cannot be created using a velocitytime graph because it does not contain any information about the object's position.
- However, the velocity-time graph does contain information about the object's displacement.
- Recall that for an object moving at a constant velocity,



- On the graph shown on the right, *v* is the height of the plotted line above the *t*-axis, while Δ*t* is the width of the shaded rectangle. The area of the rectangle, then, is *v*Δ*t*, or Δ*d*. Thus, the area under the *v*-*t* graph is equal to the object's displacement.
- The area under the v-t graph is equal to the object's displacement.



The *v*-*t* graph below shows the motion of an airplane. Find the displacement of the airplane at  $\Delta t = 1.0$  s and at  $\Delta t = 2.0$  s.





# Step 1: Analyze and Sketch the problem



The displacement is the area under the *v*-*t* graph.





The time intervals begin at t = 0.0.



Identify the known and unknown variables.

#### Known:

*v* = +75 m/s

 $\Delta t = 1.0 \text{ s}$ 

 $\Delta t = 2.0 \text{ s}$ 

Unknown:		
$\Delta d = ?$		

## Finding the Displacement from a v-t Graph



# Step 2: Solve for the Unknown



Solve for displacement during  $\Delta t = 1.0$  s.





Substitute v = +75 m/s,  $\Delta t = 1.0$  s





=+75m



Solve for displacement during  $\Delta t = 2.0$  s.





Substitute v = +75 m/s,  $\Delta t = 2.0$  s



# =(+75 m/s)(2.0 s)

=+150m

## Finding the Displacement from a v-t Graph



# Step 3: Evaluate the Answer



Are the units correct?

Displacement is measured in meters.

Do the signs make sense?

The positive sign agrees with the graph.

Is the magnitude realistic?

Moving a distance to about one football field is reasonable for an airplane.



The steps covered were:

- **Step 1:** Analyze and Sketch the Problem
  - The displacement is the area under the *v*-*t* graph.
  - The time intervals begin at t = 0.0.
- **Step 2** Solve for the Unknown
  - Solve for displacement during  $\Delta t = 1.0$  s.
  - Solve for displacement during  $\Delta t = 2.0$  s.



The steps covered were:

**Step 3:** Evaluate the Answer

# **An Alternative Expression**

- Often, it is useful to relate position, velocity, and constant acceleration without including time.
- The three equations for motion with constant acceleration are summarized in the table.

Equations of Motion for Uniform Acceleration			
Equation	Variables	Initial Conditions	
$v_{\rm f} = v_{\rm i} + \overline{a}t_{\rm f}$	t <sub>f</sub> , v <sub>f</sub> , ā	Vi	
$d_{\rm f} = d_{\rm i} + v_{\rm i}t_{\rm f} + \frac{1}{2}\overline{a}t_{\rm f}^2$	t <sub>f</sub> , d <sub>f</sub> , ā	ď <sub>i</sub> , v <sub>i</sub>	
$v_{\rm f}^2 = v_{\rm i}^2 + 2\overline{a}(d_{\rm f} - d_{\rm i})$	d <sub>f</sub> , v <sub>f</sub> , ā	ď <sub>i</sub> , V <sub>i</sub>	

# **An Alternative Expression**

Rearrange the equation  $v_f = v_i + \bar{a}t_f$ , to solve for time:

$$t_{\rm f} = rac{V_{\rm f} - V_{\rm i}}{ar{a}}$$

Rewriting  $d_f = d_i + v_i t_f + \frac{1}{2} \overline{a} t_f^2$  by substituting  $t_f$ , yields the following:

$$d_{\rm f} = d_{\rm i} + v_{\rm j} \frac{v_{\rm f} - v_{\rm i}}{\bar{a}} + \frac{1}{2} \bar{a} \left( \frac{v_{\rm f} - v_{\rm i}}{\bar{a}} \right)^2$$

# **An Alternative Expression**

This equation can be solved for the velocity,  $v_{\rm f}$ , at any time,  $t_{\rm f}$ .

$$v_{f}^{2} = v_{i}^{2} + 2 \overline{a} (d_{f} - d_{i})$$

The square of the final velocity equals the sum of the square of the initial velocity and twice the product of the acceleration and the displacement since the initial time.

# **Question 1**

A position-time graph of a bike moving with constant acceleration is shown on the right. Which statement is correct regarding the displacement of the bike?

- A. The displacement in equal time interval is constant.
- B. The displacement in equal time interval progressively increases.



- C. The displacement in equal time interval progressively decreases.
- D. The displacement in equal time interval first increases, then after reaching a particular point it decreases.
Answer 1

Answer: B

**Reason:** You will see that the slope gets steeper as time goes, which means that the displacement in equal time interval progressively gets larger and larger.

# **Question 2**

A car is moving with an initial velocity of  $v_i$  m/s. After reaching a highway, it moves with a constant acceleration of *a* m/s<sup>2</sup>, what will be the velocity ( $v_f$ ) of the car after traveling for *t* seconds?

A. 
$$v_{\rm f} = v_{\rm i} + at$$

$$\mathsf{B}. \quad v_{\mathsf{f}} = v_{\mathsf{i}} + 2at$$

C. 
$$v_f^2 = v_i^2 + 2at$$

$$\mathsf{D}. \quad v_{\mathsf{f}} = v_{\mathsf{i}} - at$$

#### Answer 2

Answer: A

**Reason:** Since 
$$a = \Delta v / \Delta t$$
  
 $v_{\rm f} - v_{\rm i} = a (t_{\rm f} - t_{\rm i})$ 

Also since car is starting from rest,  $t_i = 0$ 

Therefore  $v_f = v_i + at$  (where *t* is the total time)

#### **Question 3**

From the graph as shown on the right, of a car slowing down with a constant acceleration from initial velocity  $v_i$  to the final velocity  $v_f$ , calculate the total distance ( $\Delta d$ ) traveled by the car?



A. 
$$\Delta d = v_i (\Delta t) - \frac{1}{2} a (\Delta t)^2$$
  
B. 
$$\Delta d = v_f (\Delta t) - a (\Delta t)^2$$

c. 
$$\Delta d = v_i (\Delta t) + a (\Delta t)^2$$
  
D.  $\Delta d = v_f (\Delta t) - \frac{1}{2} a (\Delta t)^2$ 

#### Answer 3

Answer: D

Reason: Acceleration is the area under the graph.



#### Answer 3

Answer: D

Reason:

Now since 
$$(v_f - v_i) = a\Delta t$$
  
 $(v_i - v_f) = -a\Delta t$ 

Substituting in the above equation, we get  $\Delta d_{\text{triangle}} = -a(\Delta t)^2$ 

Also  $\Delta d_{\text{rectangle}} = v_{\text{f}}(\Delta t)$ 

Adding the above two equations, we can write  $\Delta d = v_{\rm f}(\Delta t) - a(\Delta t)^2$ 

#### In this section you will:

- **Define** acceleration due to gravity.
- **Solve** problems involving objects in free fall.

- After a lot of observation, Galileo concluded that, neglecting the effect of the air, all objects in free fall had the same acceleration. It didn't matter what they were made of, how much they weighed, what height they were dropped from, or whether they were dropped or thrown.
- The acceleration of falling objects, given a special symbol, g, is equal to 9.80 m/s<sup>2</sup>.
- The acceleration due to gravity is the acceleration of an object in free fall that results from the influence of Earth's gravity.



- At the top of the flight, the ball's velocity is 0 m/s. What would happen if its acceleration were also zero? Then, the ball's velocity would not be changing and would remain at 0 m/s.
- If this were the case, the ball would not gain any downward velocity and would simply hover in the air at the top of its flight.
- Because this is not the way objects tossed in the air behave on Earth, you know that the acceleration of an object at the top of its flight must not be zero. Further, because you know that the object will fall from that height, you know that the acceleration must be downward.

- Amusement parks use the concept of free fall to design rides that give the riders the sensation of free fall.
- These types of rides usually consist of three parts: the ride to the top, momentary suspension, and the plunge downward.
- When the cars are in free fall, the most massive rider and the least massive rider will have the same acceleration.



# **Question 1**

What is free fall?

#### Answer 1

Free Fall is the motion of the body when air resistance is negligible and the action can be considered due to gravity alone.

# **Question 2**

If a stone is thrown vertically upwards with a velocity of 25 m/s, what will be the velocity of the stone after 1 second?

- A. 9.8 m/s
- **B.** 15.2 m/s
- C. 25 m/s
- D. 34.8 m/s

Answer 2

Answer: B

**Reason:** Since the ball is thrown upwards, the velocity and acceleration are in opposite directions, therefore the speed of the ball decreases. After 1 s, the ball's velocity is reduced by 9.8 m/s (as acceleration due to gravity is 9.8 m/s<sup>2</sup>), so it is now traveling at 25 m/s – 9.8 m/s = 15.2 m/s.

# **Question 3**

If a 50-kg bag and a 100-kg bag are dropped from a height of 50 m. Which of the following statement is true about their acceleration? (Neglect air resistance)

- A. 100-kg bag will fall with a greater acceleration.
- B. 50-kg bag will fall with a greater acceleration.
- C. Both will fall at the same and constant rate of acceleration.
- D. Both will fall at the same rate of acceleration, which changes equally as time goes.

Answer 3

Answer: C

**Reason:** Any body falling freely towards Earth, falls with a same and constant acceleration of 9.8 m/s<sup>2</sup>. It doesn't matter how much it weighed and what height it was dropped from.

Chapter

3

# **End of Chapter**



#### **Velocity-Time Graphs**

In the graph, a pair of data points that are separated by 1 s, such as 4.00 s and 5.00 s. At 4.00 s, the car was moving at a velocity of 20.0 m/s. At 5.00 s, the car was traveling at 25.0 m/s. Thus, the car's velocity increased by 5.00 m/s in 1.00 s.







#### **Determining Acceleration from a** *v***-***t* **Graph**

Suppose you run wind sprints back and forth across the gym. You first run at 4.0 m/s toward the wall. Then, 10.0 s later, you run at 4.0 m/s away from the wall. What is your average acceleration if the positive direction is toward the wall?

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\rm f} - v_{\rm i}}{t_{\rm f} - t_{\rm i}}$$

$$= \frac{(-4.0 \text{ m/s}) - (4.0 \text{ m/s})}{10.0 \text{ s}} = \frac{-8.0 \text{ m/s}}{10.0 \text{ s}} = -0.80 \text{ m/s}^2$$



#### **Determining Acceleration from a** *v***-***t* **Graph**

- The negative sign indicates that the direction of acceleration is away from the wall. The velocity changes when the direction of motion changes, because velocity includes the direction of motion. A change in velocity results in acceleration. Thus, acceleration also is associated with a change in the direction of motion.
- There are several parallels between acceleration and velocity. Both are rates of change: acceleration is the time rate of change of velocity, and velocity is the time rate of change of position. Both acceleration and velocity have average and instantaneous forms.



#### **Position with Constant Acceleration**

On the v-t graph shown on the right, for an object moving with constant acceleration that started with an initial velocity of v<sub>i</sub>, derive the object's displacement.





#### **Position with Constant Acceleration**

- The area under the graph can be calculated by dividing it into a rectangle and a triangle. The area of the rectangle can be found by  $\Delta d_{\text{rectangle}} = v \Delta t$  and the area of the triangle can be found by .
- Because average acceleration,  $\bar{a}$ , is equal to  $\Delta v / \Delta t$ ,  $\Delta v$  can be rewritten as  $\bar{a}\Delta t$ . Substituting  $\Delta v = \bar{a}\Delta t$  into the equation for the triangle's area yields

$$\Delta d_{\text{triangle}} = \frac{1}{2} \left( \overline{a} \Delta t \right) \Delta t, \frac{1}{2} \overline{a} \left( \Delta t \right)^2$$



#### **Position with Constant Acceleration**

Solving for the total area under the graph results in the following:

$$\Delta d = \Delta d_{\text{rectangle}} + \Delta d_{\text{triangle}} = v_i (\Delta t) + \frac{1}{2} (\Delta t)^2$$

When the initial or final position of the object is known, the equation can be written as follows:

$$d_{\rm f} = d_{\rm i} = v_{\rm i} (\Delta t) + \frac{1}{2} a (\Delta t)^2 \text{ or } d_{\rm f} = d_{\rm i} + v_{\rm i} (\Delta t) + \frac{1}{2} a (\Delta t)^2$$



#### **Position with Constant Acceleration**

If the initial time,  $t_i = 0$ , the equation then becomes the following:

$$d = d_{i} + V_{i}t_{f} + \frac{1}{2}\bar{a}t_{f}^{2}$$

An object's position at a time after the initial time is equal to the sum of its initial position, the product of the initial velocity and the time, and half the product of the acceleration and the square of the time.



- Suppose the free-fall ride at an amusement park starts at rest and is in free fall for 1.5 s. What would be its velocity at the end of 1.5 s?
- Choose a coordinate system with a positive axis upward and the origin at the initial position of the car. Because the car starts at rest, v<sub>i</sub> would be equal to 0.00 m/s.



To calculate the final velocity, use the equation for velocity with constant acceleration.

$$v_{\rm f} = v_{\rm i} + \bar{a}t_{\rm f}$$

 $= 0.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(1.5 \text{ s})$ 





How far does the car fall? Use the equation for displacement when time and constant acceleration are known.

$$d_{f} = d_{i} + v_{i}t_{f} + \frac{1}{2}a_{f}^{-1}$$
  
0.00m + (0.00m/s)(1.5s) +  $\frac{1}{2}(-9.80m/s^{2})(1.5s)^{2}$ 





#### **Velocity and Acceleration**

How would you describe the sprinter's velocity and acceleration as shown on the graph?





#### Finding the Displacement from a *v*-*t* Graph

The *v*-*t* graph below shows the motion of an airplane. Find the displacement of the airplane at  $\Delta t = 1.0$  s and at  $\Delta t = 2.0$  s.



