CP Physics Chapter 5

Chapter 5

In this chapter you will:

- Represent vector quantities both graphically and algebraically.
- Use Newton's laws to analyze motion when friction is involved.
- Use Newton's laws and your knowledge of vectors to analyze motion in two dimensions.





Chapter 5: Forces in Two Dimensions

Section 5.1: Vectors

Section 5.2: Friction

Section 5.3: Force and Motion in Two Dimensions



In this section you will:

- Evaluate the sum of two or more vectors in two dimensions graphically.
- **Determine** the components of vectors.
- Solve for the sum of two or more vectors algebraically by adding the components of the vectors.

- The process for adding vectors works even when the vectors do not point along the same straight line.
- If you are solving one of these two-dimensional problems graphically, you will need to use a protractor, both to draw the vectors at the correct angle and also to measure the direction and magnitude of the resultant vector.
- You can add vectors by placing them tip-to-tail and then drawing the resultant of the vector by connecting the tail of the first vector to the tip of the second vector.

- The figure below shows the two forces in the free-body diagram.
- If you move one of the vectors so that its tail is at the same place as the tip of the other vector, its length and direction do not change.





- If you move a vector so that its length and direction are unchanged, the vector is unchanged.
- You can draw the resultant vector pointing from the tail of the first vector to the tip of the last vector and measure it to obtain its magnitude.
- Use a protractor to measure the direction of the resultant vector.





- Sometimes you will need to use trigonometry to determine the length or direction of resultant vectors.
- If you are adding together two vectors at right angles, vector A pointing north and vector B pointing east, you could use the Pythagorean theorem to find the magnitude of the resultant, R.
- If vector A is at a right angle to vector B, then the sum of the squares of the magnitudes is equal to the square of the magnitude of the resultant vector.



If two vectors to be added are at an angle other than 90°, then you can use the law of cosines or the law of sines.

Law of cosines: $R^2 = A^2 + B^2 - 2AB \cos \theta$

The square of the magnitude of the resultant vector is equal to the sum of the magnitude of the squares of the two vectors, minus two times the product of the magnitudes of the vectors, multiplied by the cosine of the angle between them.

Law of sines:



The magnitude of the resultant, divided by the sine of the angle between two vectors, is equal to the magnitude of one of the vectors divided by the angle between that component vector and the resultant vector.

Find the magnitude of the sum of a 15-km displacement and a 25-km displacement when the angle between them is 90° and when the angle between them is 135°.

Finding the Magnitude of the Sum of Two Vectors



Step 1: Analyze and Sketch the Problem



Sketch the two displacement vectors, **A** and **B**, and the angle between them.





Identify the known and unknown variables.



Known:	Unknown:
A = 25 km	<i>R</i> = ?
<i>B</i> = 15 km	
$\theta_1 = 90^\circ$	
θ_2 = 135 °	

Finding the Magnitude of the Sum of Two Vectors



Step 2: Solve for the Unknown



When the angle is 90°, use the Pythagorean theorem to find the magnitude of the resultant vector.







Substitute A = 25 km, B = 15 km







When the angle does not equal 90°, use the law of cosines to find the magnitude of the resultant vector.

 $R^2 = A^2 + B^2 - 2AB(\cos\theta_2)$

$$R = \sqrt{A^2 + B^2 - 2AB(\cos\theta_2)}$$



Substitute A = 25 km, B = 15 km, $\theta_2 = 135^{\circ}$.





Finding the Magnitude of the Sum of Two Vectors



Step 3: Evaluate the Answer



Are the units correct?

Each answer is a length measured in kilometers.

Do the signs make sense?

The sums are positive.

Finding the Magnitude of the Sum of Two Vectors



Are the magnitudes realistic?

The magnitudes are in the same range as the two combined vectors, but longer. This is because each resultant is the side opposite an obtuse angle. The second answer is larger than the first, which agrees with the graphical representation.



The steps covered were:

- **Step 1:** Analyze and sketch the Problem
 - Sketch the two displacement vectors, *A* and *B*, and the angle between them.
- **Step 2:** Solve for the Unknown
 - When the angle is 90°, use the Pythagorean theorem to find the magnitude of the resultant vector.
 - When the angle does not equal 90°, use the law of cosines to find the magnitude of the resultant vector.



The steps covered were:

Step 3: Evaluate the answer

Components of Vectors

- Choosing a coordinate system, such as the one shown below, is similar to laying a grid drawn on a sheet of transparent plastic on top of a vector problem.
- You have to choose where to put the center of the grid (the origin) and establish the directions in which the axes point.



Components of Vectors

- When the motion you are describing is confined to the surface of Earth, it is often convenient to have the x-axis point east and the y-axis point north.
- When the motion involves an object moving through the air, the positive x-axis is often chosen to be horizontal and the positive y-axis vertical (upward).
- If the motion is on a hill, it's convenient to place the positive xaxis in the direction of the motion and the y-axis perpendicular to the x-axis.

Component Vectors



Click image to view movie.

- Two or more vectors (A, B, C, etc.) may be added by first resolving each vector into its x- and y-components.
- The *x*-components are added to form the *x*-component of the resultant: $R_x = A_x + B_x + C_x$.
- Similarly, the *y*-components are added to form the *y*-component of the resultant:

$$R_{\rm y}=A_{\rm y}+B_{\rm y}+C_{\rm y}.$$



Because R_x and R_y are at a right angle (90°), the magnitude of the resultant vector can be calculated using the Pythagorean theorem, $R^2 = R_x^2 + R_y^2$.





To find the angle or direction of the resultant, recall that the tangent of the angle that the vector makes with the x-axis is give by the following.

Angle of the resultant vector



The angle of the resultant vector is equal to the inverse tangent of the quotient of the *y*-component divided by the *x*-component of the resultant vector.

- You can find the angle by using the \tan^{-1} key on your calculator.
- Note that when tan $\theta > 0$, most calculators give the angle between 0° and 90°, and when tan $\theta < 0$, the angle is reported to be between 0° and -90° .
- You will use these techniques to resolve vectors into their components throughout your study of physics.
- Resolving vectors into components allows you to analyze complex systems of vectors without using graphical methods.

Question 1

Jeff moved 3 m due north, and then 4 m due west to his friends house. What is the displacement of Jeff?

- A. 3 + 4 m
- **B.** 4 3 m
- **C.** 3² + 4² m
- D. 5 m

Answer 1

Answer: D

Reason: When two vectors are at right angles to each other as in this case, we can use the Pythagorean theorem of vector addition to find the magnitude of resultant, *R*.



Answer 1

Answer: D

Reason: Pythagorean theorem of vector addition states

If vector A is at right angle to vector B then the sum of squares of magnitudes is equal to square of magnitude of resultant vector.

That is, $R^2 = A^2 + B^2$

 $\therefore R^2 = (3 \text{ m})^2 + (4 \text{ m})^2 = 5 \text{ m}$

Question 2

Calculate the resultant of three vectors *A*, *B*, *C* as shown in the figure.

$$(A_x = B_x = C_x = A_y = C_y = 1$$
 units
and $B_y = 2$ units)



- A. 3 + 4 units
- **B.** $3^2 + 4^2$ units

C.
$$4^2 - 3^2$$
 units
D. $\sqrt{4^2 + 3^2}$ units

Answer 2

Answer: D

Reason: Add the *x*-components to form, $R_x = A_x + B_x + C_x$.

Add the *y*-components to form, $R_y = A_y + B_y + C_y$.


Answer 2

Answer: D

Reason: Since R_x and R_y are perpendicular to each other we can apply Pythagoras theorem of vector addition:

$$R^2 = R_x^2 + R_y^2$$

 $\therefore R = \sqrt{4^2 + 3^2}$ units

Question 3

If a vector **B** is resolved into two components B_x and B_y and if θ is angle that vector **B** makes with the positive direction of *x*-axis, Using which of the following formula can you calculate the components of vector **B**?

A.
$$B_x = B \sin \theta$$
, $B_y = B \cos \theta$

B.
$$B_x = B \cos \theta$$
, $B_y = B \sin \theta$

C.
$$B_x = \frac{B}{\cos \theta}, B_y = \frac{B}{\sin \theta}$$

D. $B_x = \frac{B}{\sin \theta}, B_y = \frac{B}{\cos \theta}$

Answer 3

Answer: B

Reason: The components of vector 'B' are calculated using the equation stated below.





Answer 3

Answer: B

Reason:

Also cos θ =	Adjacent side
	hypotenuse

$$\therefore \cos \theta = \frac{B_x}{B}$$



In this section you will:

- **Define** the friction force.
- **Distinguish** between static and kinetic friction.

- Push your hand across your desktop and feel the force called friction opposing the motion.
- There are two types of friction, and both always oppose motion.
- When you push a book across the desk, it experiences a type of friction that acts on moving objects.
- This force is known as kinetic friction, and it is exerted on one surface by another when the two surfaces rub against each other because one or both of them are moving.

- To understand the other kind of friction, imagine trying to push a heavy couch across the floor. You give it a push, but it does not move.
- Because it does not move, Newton's laws tell you that there must be a second horizontal force acting on the couch, one that opposes your force and is equal in size.
- This force is static friction, which is the force exerted on one surface by another when there is no motion between the two surfaces.

- You might push harder and harder, as shown in the figure below, but if the couch still does not move, the force of friction must be getting larger.
- This is because the static friction force acts in response to other forces.





- Finally, when you push hard enough, as shown in the figure below, the couch will begin to move.
- Evidently, there is a limit to how large the static friction force can be. Once your force is greater than this maximum static friction, the couch begins moving and kinetic friction begins to act on it instead of static friction.



- Frictional force depends on the materials that the surfaces are made of.
- For example, there is more friction between skis and concrete than there is between skis and snow.
- The normal force between the two objects also matters. The harder one object is pushed against the other, the greater the force of friction that results.

- If you pull a block along a surface at a constant velocity, according to Newton's laws, the frictional force must be equal and opposite to the force with which you pull.
- You can pull a block of known mass along a table at a constant velocity and use a spring scale, as shown in the figure, to measure the force that you exert.
- You can then stack additional blocks on the block to increase the normal force and repeat the measurement.





- Plotting the data will yield a graph like the one shown here. There is a direct proportion between the kinetic friction force and the normal force.
- The different lines correspond to dragging the block along different surfaces.
- Note that the line corresponding to the sandpaper surface has a steeper slope than the line for the highly polished table.



Normal force

- You would expect it to be much harder to pull the block along sandpaper than along a polished table, so the slope must be related to the magnitude of the resulting frictional force.
- The slope of this line, designated μ_k , is called the **coefficient of kinetic friction** between the two surfaces and relates the frictional force to the normal force, as shown below.

Kinetic friction force

$$F_{\rm f,\,kinetic} = \mu_{\rm k} F_{\rm N}$$

The kinetic friction force is equal to the product of the coefficient of the kinetic friction and the normal force.

- The maximum static friction force is related to the normal force in a similar way as the kinetic friction force.
- The static friction force acts in response to a force trying to cause a stationary object to start moving. If there is no such force acting on an object, the static friction force is zero.
- If there is a force trying to cause motion, the static friction force will increase up to a maximum value before it is overcome and motion starts.

Static Friction Force



- The static friction force is less than or equal to the product of the coefficient of the static friction and the normal force.
- In the equation for the maximum static friction force, μ_s is the **coefficient of static friction** between the two surfaces, and $\mu_s F_N$ is the maximum static friction force that must be overcome before motion can begin.

- Note that the equations for the kinetic and maximum static friction forces involve only the magnitudes of the forces.
- The forces themselves, $F_{\rm f}$ and $F_{\rm N}$, are at right angles to each other. The table here shows coefficients of friction between various surfaces.
- Although all the listed coefficients are less than 1.0, this does not mean that they must always be less than 1.0.

Typical Coefficients of Friction		
Surface	μ_{s}	μ_{k}
Rubber on dry concrete	0.80	0.65
Rubber on wet concrete	0.60	0.40
Wood on wood	0.50	0.20
Steel on steel (dry)	0.78	0.58
Steel on steel (with oil)	0.15	0.06

You push a 25.0 kg wooden box across a wooden floor at a constant speed of 1.0 m/s. How much force do you exert on the box?

Balanced Friction Forces



Step 1: Analyze and Sketch the Problem



Identify the forces and establish a coordinate system.





Draw a motion diagram indicating constant v and a = 0.





Draw the free-body diagram.





Identify the known and unknown variables.

Known:

Unknown:

m = 25.0 kg

v = 1.0 m/s

 $a = 0.0 \text{ m/s}^2$

 $\mu_{\rm k} = 0.20$

Balanced Friction Forces



Step 2: Solve for the Unknown



The normal force is in the *y*-direction, and there is no acceleration.

$$F_{\rm N} = F_{\rm g}$$

= mg



Substitute $m = 25.0 \text{ kg}, g = 9.80 \text{ m/s}^2$

$$F_{\rm N} = 25.0 \text{ kg}(9.80 \text{ m/s}^2)$$

= 245 N



The pushing force is in the *x*-direction; *v* is constant, thus there is no acceleration.

 $F_{\rm p} = \mu_{\rm k} mg$



Substitute $\mu_{\rm k}$ = 0.20, *m* = 25.0 kg, *g* = 9.80 m/s²

$$F_{\rm p} = (0.20)(25.0 \text{ kg})(9.80 \text{ m/s}^2)$$

= 49 N





Are the units correct?

Performing dimensional analysis on the units verifies that force is measured in kg·m/s² or N.

Does the sign make sense?

The positive sign agrees with the sketch.

Is the magnitude realistic?

The force is reasonable for moving a 25.0 kg box.



The steps covered were:

- **Step 1:** Analyze and sketch the problem
 - Identify the forces and establish a coordinate system.
 - Draw a motion diagram indicating constant v and a = 0.
 - Draw the free-body diagram.



The steps covered were:

- **Step 2:** Solve for the unknown
 - The normal force is in the *y*-direction, and there is no acceleration.
 - The pushing force is in the *x*-direction; *v* is constant, thus there is no acceleration.
- **Step 3:** Evaluate the answer

Question 1

Define friction force.

Answer 1

A force that opposes motion is called friction force. There are two types of friction force:

- 1) Kinetic friction—exerted on one surface by another when the surfaces rub against each other because one or both of them are moving.
- 2) Static friction—exerted on one surface by another when there is no motion between the two surfaces.

Question 2

Juan tried to push a huge refrigerator from one corner of his home to another, but was unable to move it at all. When Jason accompanied him, they where able to move it a few centimeter before the refrigerator came to rest. Which force was opposing the motion of the refrigerator?

- A. Static friction
- B. Kinetic friction
- C. Before the refrigerator moved, static friction opposed the motion. After the motion, kinetic friction opposed the motion.
- D. Before the refrigerator moved, kinetic friction opposed the motion. After the motion, static friction opposed the motion.

Answer 2

Answer: C

Reason: Before the refrigerator started moving, the static friction, which acts when there is no motion between the two surfaces, was opposing the motion. But static friction has a limit. Once the force is greater than this maximum static friction, the refrigerator begins moving. Then, kinetic friction, the force acting between the surfaces in relative motion, begins to act instead of static friction.

Question 3

On what does a friction force depends?

- A. The material that the surface are made of
- B. The surface area
- C. Speed of the motion
- D. The direction of the motion

Answer 3

Answer: A

Reason: The materials that the surfaces are made of play a role. For example, there is more friction between skis and concrete than there is between skis and snow.
Question 4

A player drags three blocks in a drag race, a 50-kg block, a 100-kg block, and a 120-kg block with the same velocity. Which of the following statement is true about the kinetic friction force acting in each case?

- A. Kinetic friction force is greater while dragging 50-kg block.
- B. Kinetic friction force is greater while dragging 100-kg block.
- C. Kinetic friction force is greater while dragging 120-kg block.
- D. Kinetic friction force is same in all the three cases.

Answer 4

Answer: C

Reason: Kinetic friction force is directly proportional to the normal force, and as the mass increases the normal force also increases. Hence, the kinetic friction force will hit its limit while dragging the maximum weight.

In this section you will:

- Determine the force that produces equilibrium when three forces act on an object.
- Analyze the motion of an object on an inclined plane with and without friction.

- Now you will use your skill in adding vectors to analyze situations in which the forces acting on an object are at angles other than 90°.
- Recall that when the net force on an object is zero, the object is in equilibrium.
- According to Newton's laws, the object will not accelerate because there is no net force acting on it; an object in equilibrium is motionless or moves with constant velocity.

- It is important to realize that equilibrium can occur no matter how many forces act on an object. As long as the resultant is zero, the net force is zero and the object is in equilibrium.
- The figure here shows three forces exerted on a point object. What is the net force acting on the object?
- Remember that vectors may be moved if you do not change their direction (angle) or length.



- The figure here shows the addition of the three forces, A, B, and C.
- Note that the three vectors form a closed triangle.
- There is no net force; thus, the sum is zero and the object is in equilibrium.



- Suppose that two forces are exerted on an object and the sum is not zero.
- How could you find a third force that, when added to the other two, would add up to zero, and therefore cause the object to be in equilibrium?
- To find this force, first find the sum of the two forces already being exerted on the object.
- This single force that produces the same effect as the two individual forces added together, is called the resultant force.

- The force that you need to find is one with the same magnitude as the resultant force, but in the opposite direction.
- A force that puts an object in equilibrium is called an equilibrant.

- The figure below illustrates the procedure for finding the equilibrant for two vectors.
- This general procedure works for any number of vectors.



Motion Along an Inclined Plane



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Motion Along an Inclined Plane

- Because an object's acceleration is usually parallel to the slope, one axis, usually the x-axis, should be in that direction.
- The y-axis is perpendicular to the x-axis and perpendicular to the surface of the slope.
- With this coordinate system, there are two forces—normal and frictional forces. These forces are in the direction of the coordinate axes. However, the weight is not.
- This means that when an object is placed on an inclined plane, the magnitude of the normal force between the object and the plane will usually not be equal to the object's weight.

Motion Along an Inclined Plane

- You will need to apply Newton's laws once in the x-direction and once in the y-direction.
- Because the weight does not point in either of these directions, you will need to break this vector into its x- and y-components before you can sum your forces in these two directions.

Question 1

If three forces **A**, **B**, and **C** are exerted on an object as shown in the following figure, what is the net force acting on the object? Is the object in equilibrium?



Answer 1

We know that vectors can be moved if we do not change their direction and length.

The three vectors *A*, *B*, and *C* can be moved (rearranged) to form a closed triangle.

Since the three vectors form a closed triangle, there is no net force. Thus, the sum is zero and the object is in equilibrium. An object is in equilibrium when all the forces add up to zero.



Question 2

How do you decide the coordinate system when the motion is along a slope? Is the normal force between the object and the plane the object's weight?

Answer 2

An object's acceleration is usually parallel to the slope. One axis, usually the x-axis, should be in that direction. The y-axis is perpendicular to the x-axis and perpendicular to the surface of the slope. With these coordinate systems, you have two forces—the normal force and the frictional force. Both are in the direction of the coordinate axes. However, the weight is not. This means that when an object is placed on an inclined plane, the magnitude of the normal force between the object and the plane will usually not be equal to the object's weight.

Question 3

A skier is coming down the hill. What are the forces acting parallel to the slope of the hill?

- A. Normal force and weight of the skier.
- B. Frictional force and component of weight of the skier along the slope.
- C. Normal force and frictional force.
- D. Frictional force and weight of the skier.

Answer 3

Answer: B

Reason: The component of the weight of the skier will be along the slope, which is also the direction of the skier's motion. The frictional force will be in the opposite direction from the direction of motion of the skier.

Chapter 5

End of Chapter



Finding the Magnitude of the Sum of Two Vectors

Find the magnitude of the sum of a 15-km displacement and a 25-km displacement when the angle between them is 90° and when the angle between them is 135°.



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Balanced Friction Forces

You push a 25.0 kg wooden box across a wooden floor at a constant speed of 1.0 m/s. How much force do you exert on the box?



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