CP Physics
Chapter 6

Chapter
6

In this chapter you will:

- Use Newton's laws and your knowledge of vectors to analyze motion in two dimensions.
- Solve problems dealing with projectile and circular motion.
- Solve relative velocity problems.

Chapter 6: Motion in Two Dimensions

In this section you will:

- **Recognize** that the vertical and horizontal motions of a projectile are independent.
- **Relate** the height, time in the air, and initial vertical velocity of a projectile using its vertical motion, and then determine the range using the horizontal motion.
- **Explain** how the trajectory of a projectile depends upon the frame of reference from which it is observed.

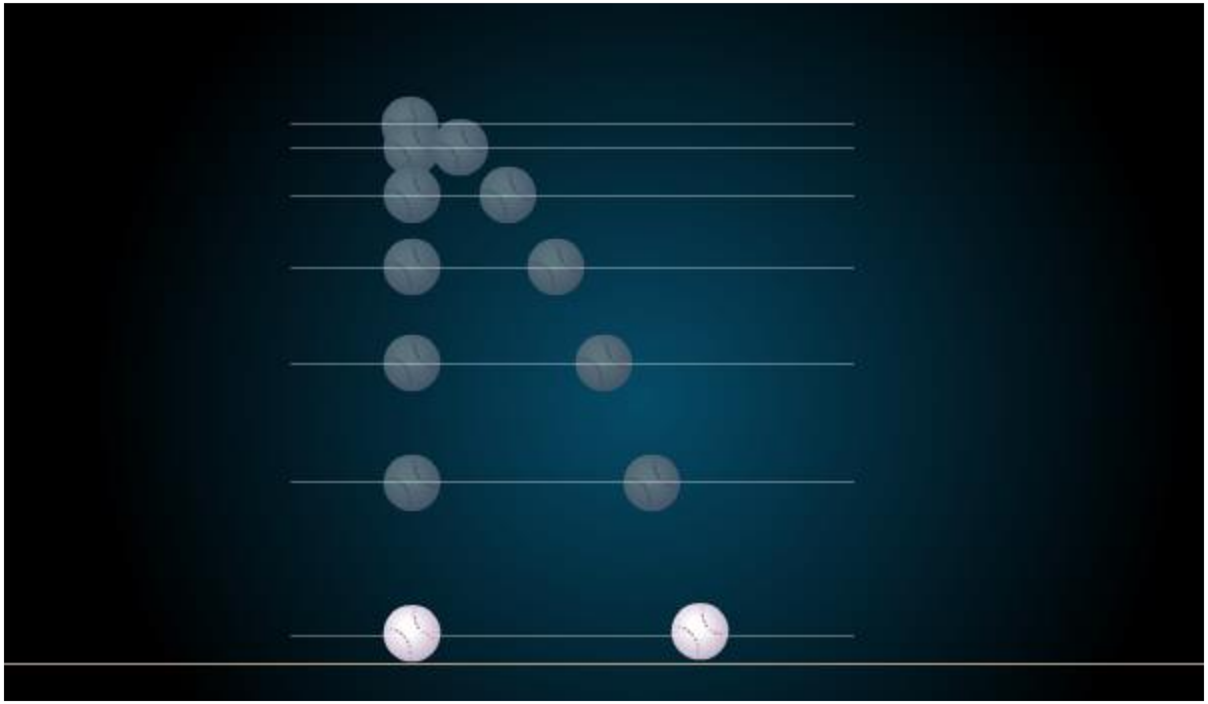
Projectile Motion

- If you observed the movement of a golf ball being hit from a tee, a frog hopping, or a free throw being shot with a basketball, you would notice that all of these objects move through the air along similar paths, as do baseballs, arrows, and bullets.
- Each path is a curve that moves upward for a distance, and then, after a time, turns and moves downward for some distance.
- You may be familiar with this curve, called a parabola, from math class.

Projectile Motion

- An object shot through the air is called a **projectile**.
- A projectile can be a football, a bullet, or a drop of water.
- You can draw a free-body diagram of a launched projectile and identify all the forces that are acting on it.
- No matter what the object is, after a projectile has been given an initial thrust, if you ignore air resistance, it moves through the air only under the force of gravity.
- The force of gravity is what causes the object to curve downward in a parabolic flight path. Its path through space is called its **trajectory**.

Independence of Motion in Two Dimensions



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Projectiles Launched at an Angle

- When a projectile is launched at an angle, the initial velocity has a vertical component as well as a horizontal component.

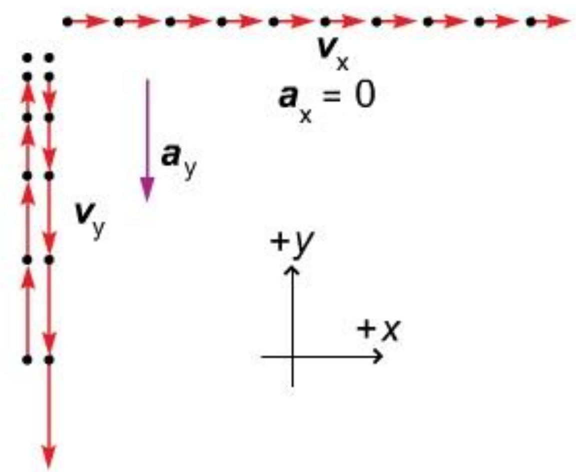
- If the object is launched upward, like a ball tossed straight up in the air, it rises with slowing speed, reaches the top of its path, and descends with increasing speed.

Projectiles Launched at an Angle

- The adjoining figure shows the separate vertical- and horizontal-motion diagrams for the trajectory of the ball.

- At each point in the vertical direction, the velocity of the object as it is moving upward has the same magnitude as when it is moving downward.

- The only difference is that the directions of the two velocities are opposite.



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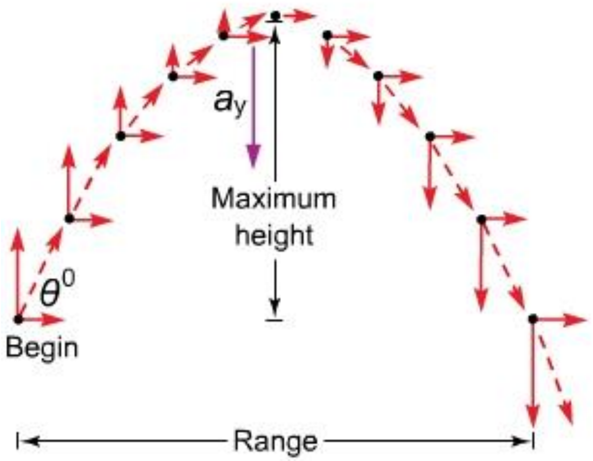
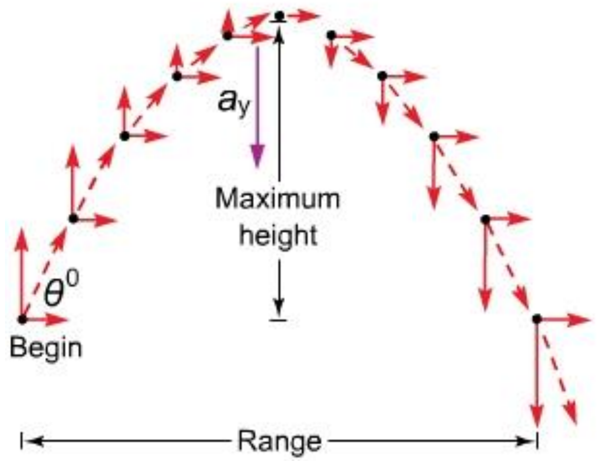
Projectiles Launched at an Angle

- The adjoining figure defines two quantities associated with a trajectory.
- One is the maximum height, which is the height of the projectile when the vertical velocity is zero and the projectile has only its horizontal-velocity component.



Projectiles Launched at an Angle

- The other quantity depicted is the range, *R*, which is the horizontal distance that the projectile travels.
- Not shown is the flight time, which is how much time the projectile is in the air.
- For football punts, flight time often is called hang time.



View Movie

Section

6.1

The Flight of a Ball

A ball is launched at 4.5 m/s at 66° above the horizontal. What are the maximum height and flight time of the ball?

The Flight of a Ball

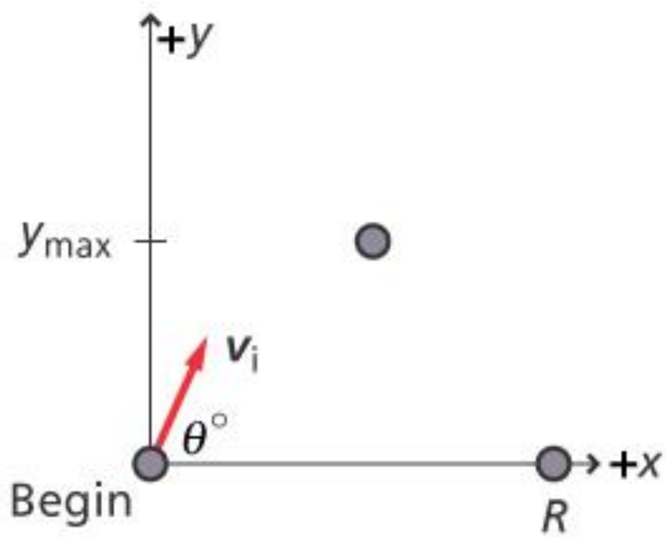
View Question

Step 1: Analyze and Sketch the Problem

The Flight of a Ball

? View Question

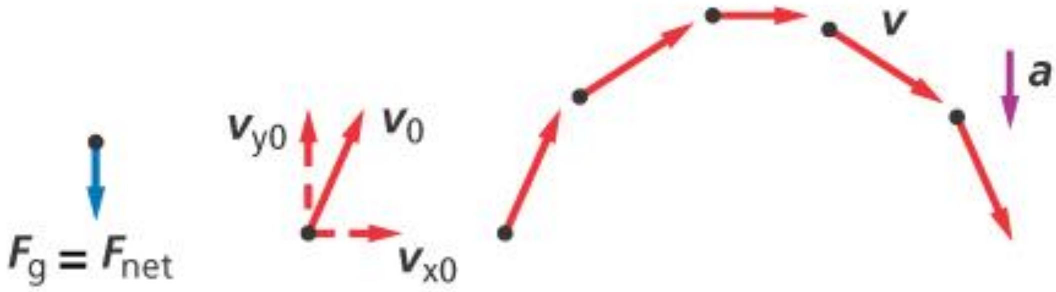
Establish a coordinate system with the initial position of the ball at the origin.



The Flight of a Ball

? View Question

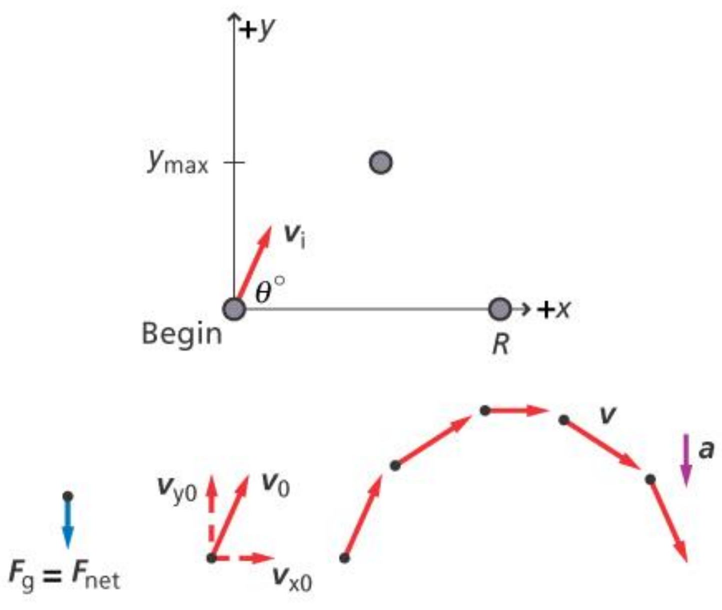
Show the positions of the ball at the beginning, at the maximum height, and at the end of the flight.



The Flight of a Ball

View Question

Draw a motion diagram showing $\boldsymbol{v}$, $\boldsymbol{a}$, and $\boldsymbol{F}_{net}$.



Known:

$y_i = 0.0$ m

$\theta_i = 66°$

$v_i = 4.5$ m/s

$a_y = -g$

Unknown:

$y_{max} = ?$

$t = ?$

The Flight of a Ball

View Question

Step 2: Solve for the Unknown

The Flight of a Ball

View Question

Find the *y*-component of v_i.

$$v_{yi} = v_i(\sin\theta_i)$$

The Flight of a Ball

? View Question

Substitute $v_i = 4.5\ \text{m/s}$, $\theta_i = 66°$

$$v_{yi} = (4.5\ \text{m/s})(\sin 66°)$$
$$= 4.1\ \text{m/s}$$

The Flight of a Ball

View Question

Find an expression for time.

$$v_y = v_{yi} + a_y t$$

The Flight of a Ball

View Question

Substitute $a_y = -g$

$$v_y = v_{yi} - gt$$

The Flight of a Ball

View Question

Solve for *t*.

$$t = \frac{v_{yi} - v_y}{g}$$

The Flight of a Ball

View Question

Solve for the maximum height.

$$y_{max} = y_i + v_{yi}t + \frac{1}{2}at^2$$

The Flight of a Ball

View Question

Substitute $t = \frac{v_{yi} - v_y}{g}, a = -g$

$$y_{max} = y_i + v_{yi}\left(\frac{v_{yi} - v_y}{g}\right) + \frac{1}{2}(-g)\left(\frac{v_{yi} - v_y}{g}\right)^2$$

The Flight of a Ball

View Question

Substitute y_i = 0.0 m, v_{yi} = 4.1 m/s, v_y = 0.0 m/s at y_{max}, g = 9.80 m/s^2

$$y_{max} = 0.0\,\text{m} + (4.1\,\text{m/s})\left(\frac{4.1\,\text{m/s} - 0.0\,\text{m/s}}{9.80\,\text{m/s}^2}\right) + \frac{1}{2}(-9.80\,\text{m/s}^2)\left(\frac{4.1\,\text{m/s} - 0.0\,\text{m/s}}{9.80\,\text{m/s}^2}\right)^2$$

$$= 0.86\,\text{m}$$

The Flight of a Ball

View Question

Solve for the time to return to the launching height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}at^2$$

The Flight of a Ball

View Question

Substitute y_f = 0.0 m, y_i = 0.0 m, $a = -g$

$$0.0\ \text{m} = 0.0\ \text{m} + v_{yi}t - \frac{1}{2}gt^2$$

The Flight of a Ball

? View Question

Use the quadratic formula to solve for *t*.

$$t = \frac{-v_{yi} \pm \sqrt{v_{yi}^{2} - 4\left(-\frac{1}{2}g\right)(0.0\ \mathrm{m})}}{2\left(-\frac{1}{2}g\right)}$$

$$= \frac{-v_{yi} \pm v_{yi}}{-g}$$

The Flight of a Ball

View Question

0 is the time the ball left the launch, so use this solution.

$$t = \frac{2v_{yi}}{g}$$

The Flight of a Ball

View Question

Substitute v_{yi} = 4.1 m/s, g = 9.80 m/s^2

$$t = \frac{(2)(4.1\ \text{m/s})}{(9.80\ \text{m/s}^2)}$$

$$= 0.84\ \text{s}$$

The Flight of a Ball

View Question

Step 3: Evaluate the Answer

The Flight of a Ball

View Question

- **Are the units correct?**

  Dimensional analysis verifies that the units are correct.

- **Do the signs make sense?**

  All should be positive.

- **Are the magnitudes realistic?**

  0.84 s is fast, but an initial velocity of 4.5 m/s makes this time reasonable.

The Flight of a Ball

View Question

The steps covered were:

- **Step 1:** Analyze and Sketch the Problem
  - Establish a coordinate system with the initial position of the ball at the origin.
  - Show the positions of the ball at the beginning, at the maximum height, and at the end of the flight.
  - Draw a motion diagram showing $\boldsymbol{v}$, $\boldsymbol{a}$, and $\boldsymbol{F}_{net}$.

The Flight of the Ball

View Question

The steps covered were:

- **Step 2:** Solve for the Unknown
  - Find the y-component of v_i.
  - Find an expression for time.
  - Solve for the maximum height.
  - Solve for the time to return to the launching height.
- **Step 3:** Evaluate the Answer

Trajectories Depend upon the Viewer

- The path of the projectile, or its trajectory, depends upon who is viewing it.
- Suppose you toss a ball up and catch it while riding in a bus. To you, the ball would seem to go straight up and straight down.
- But an observer on the sidewalk would see the ball leave your hand, rise up, and return to your hand, but because the bus would be moving, your hand also would be moving. The bus, your hand, and the ball would all have the same horizontal velocity.

Trajectories Depend upon the Viewer

- So far, air resistance has been ignored in the analysis of projectile motion.
- While the effects of air resistance are very small for some projectiles, for others, the effects are large and complex. For example, dimples on a golf ball reduce air resistance and maximize its range.
- The force due to air resistance does exist and it can be important.

Question 1

A boy standing on a balcony drops one ball and throws another with an initial horizontal velocity of 3 m/s. Which of the following statements about the horizontal and vertical motions of the balls is correct? (Neglect air resistance.)

A. The balls fall with a constant vertical velocity and a constant horizontal acceleration.

B. The balls fall with a constant vertical velocity as well as a constant horizontal velocity.

C. The balls fall with a constant vertical acceleration and a constant horizontal velocity.

D. The balls fall with a constant vertical acceleration and an increasing horizontal velocity.

Answer 1

Answer: C

Reason: The vertical and horizontal motions of a projectile are independent. The only force acting on the two balls is force due to gravity. Because it acts in the vertical direction, the balls accelerate in the vertical direction. The horizontal velocity remains constant throughout the flight of the balls.

Question 2

Which of the following conditions is met when a projectile reaches its maximum height?

A. Vertical component of the velocity is zero.

B. Vertical component of the velocity is maximum.

C. Horizontal component of the velocity is maximum.

D. Acceleration in the vertical direction is zero.

Answer 2

Answer: A

Reason: The maximum height is the height at which the object stops its upward motion and starts falling down, i.e. when the vertical component of the velocity becomes zero.

Question 3

Suppose you toss a ball up and catch it while riding in a bus. Why does the ball fall in your hands rather than falling at the place where you tossed it?

Answer 3

Trajectory depends on the frame of reference.

For an observer on the ground, when the bus is moving, your hand is also moving with the same velocity as the bus, i.e. the bus, your hand, and the ball will have the same horizontal velocity. Therefore, the ball will follow a trajectory and fall back in your hands.

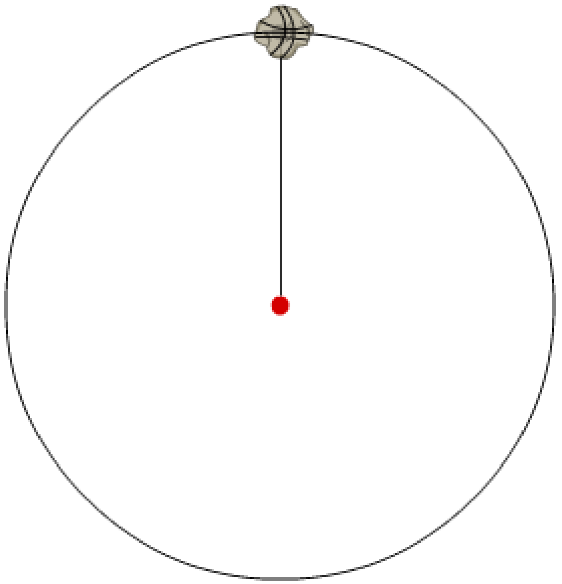
In this section you will:

- **Explain** why an object moving in a circle at a constant speed is accelerated.
- **Describe** how centripetal acceleration depends upon the object’s speed and the radius of the circle.
- **Identify** the force that causes centripetal acceleration.

Section

6.2

Describing Circular Motion



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Centripetal Acceleration

- The angle between position vectors r_1 and r_2 is the same as that between velocity vectors v_1 and v_2.

- Thus, $\Delta r/r = \Delta v/v$. The equation does not change if both sides are divided by Δt.

$$\frac{\Delta r}{r\Delta t} = \frac{\Delta v}{v\Delta t}$$

- However, $v = \Delta r/\Delta t$ and $a = \Delta v/\Delta t$

$$\frac{1}{r}\left(\frac{\Delta r}{\Delta t}\right) = \frac{1}{v}\left(\frac{\Delta v}{\Delta t}\right)$$

- Replacing $v = \Delta r/\Delta t$ in the left-hand side and $g = \Delta v/\Delta t$ in the right-hand side gives the following equation:

$$\frac{v}{r} = \frac{a}{v}$$

Centripetal Acceleration

- Solve the equation for acceleration and give it the special symbol a_c, for centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

- Centripetal acceleration always points to the center of the circle. Its magnitude is equal to the square of the speed, divided by the radius of motion.

Centripetal Acceleration

- One way of measuring the speed of an object moving in a circle is to measure its period, T, the time needed for the object to make one complete revolution.
- During this time, the object travels a distance equal to the circumference of the circle, $2\pi r$. The object's speed, then, is represented by $v = 2\pi r/T$.

$$a_c = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

Centripetal Acceleration

- Because the acceleration of an object moving in a circle is always in the direction of the net force acting on it, there must be a net force toward the center of the circle. This force can be provided by any number of agents.

- When a hammer thrower swings the hammer, as in the adjoining figure**,** the force is the tension in the chain attached to the massive ball.

Centripetal Acceleration

- When an object moves in a circle, the net force toward the center of the circle is called the **centripetal force**.
- To analyze centripetal acceleration situations accurately, you must identify the agent of the force that causes the acceleration. Then you can apply Newton's second law for the component in the direction of the acceleration in the following way.

Newton's Second Law for Circular Motion $F_{net} = ma_c$

- The net centripetal force on an object moving in a circle is equal to the object's mass times the centripetal acceleration.

Centripetal Acceleration

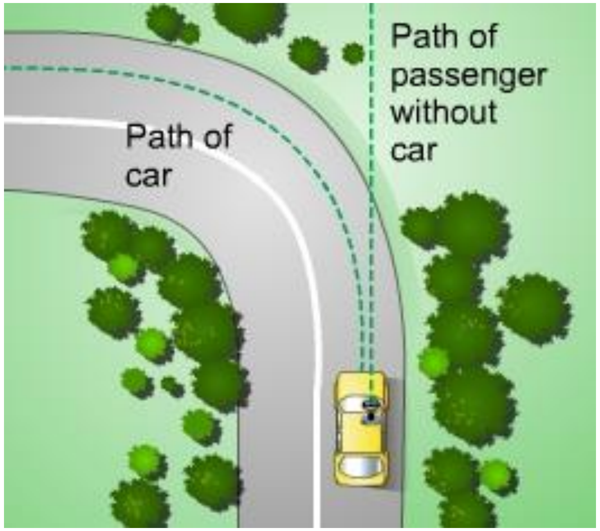
- When solving problems, it is useful to choose a coordinate system with one axis in the direction of the acceleration.
- For circular motion, the direction of the acceleration is always toward the center of the circle.
- Rather than labeling this axis *x* or *y*, call it *c*, for centripetal acceleration. The other axis is in the direction of the velocity, tangent to the circle. It is labeled *tang* for tangential.
- Centripetal force is just another name for the net force in the centripetal direction. It is the sum of all the real forces, those for which you can identify agents that act along the centripetal axis.

Example

A Nonexistent Force

- According to Newton's first law, you will continue moving with the same velocity unless there is a net force acting on you.

- The passenger in the car would continue to move straight ahead if it were not for the force of the door acting in the direction of the acceleration.



View Movie

- The so-called centrifugal, or outward force, is a fictitious, nonexistent force.

?

Question 1

Explain why an object moving in a circle at a constant speed is accelerated.

Answer 1

Because acceleration is the rate of change of velocity, the object accelerates due to the change in the direction of motion and not speed.

Section
6.2

Question 2

What is the relationship between the magnitude of centripetal acceleration (a_c) and an object's speed (v)?

A. $a_c \propto v$

B. $a_c \propto \frac{1}{v^2}$

C. $a_c \propto v^2$

D. $a_c \propto \frac{1}{v}$

Answer 2

Answer: C

Reason: From the equation for centripetal acceleration,

$$a_c = \frac{v^2}{r}$$

That is, centripetal acceleration always points to the center of the circle. Its magnitude is equal to the square of the speed divided by the radius of the motion.

Question 3

What is the direction of the velocity vector of an accelerating object?

A. Toward the center of the circle.

B. Away from the center of the circle.

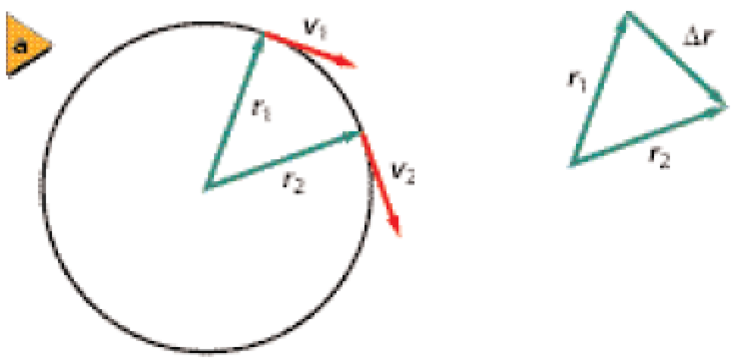
C. Along the circular path.

D. Tangent to the circular path.

Answer 3

Answer: D

Reason: The displacement, $\Delta \boldsymbol{r}$, of an object in a circular motion divided by the time interval in which the displacement occurs is the object's average velocity during that time interval. If you notice the picture below, $\Delta \boldsymbol{r}$ is in the direction of tangent to the circle and, therefore, is the velocity.



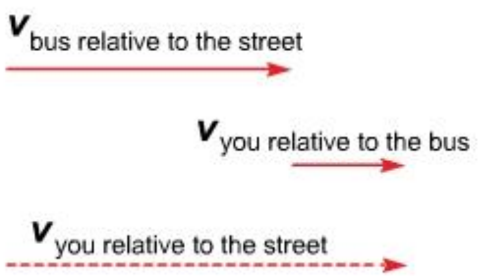
In this section you will:

- **Analyze** situations in which the coordinate system is moving.
- **Solve** relative-velocity problems.

Relative Velocity

- Suppose you are in a school bus that is traveling at a velocity of 8 m/s in a positive direction. You walk with a velocity of 3 m/s toward the front of the bus.

- If the bus is traveling at 8 m/s, this means that the velocity of the bus is 8 m/s, as measured by your friend in a coordinate system fixed to the road.

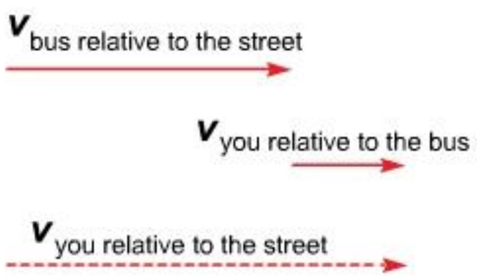


View Movie

Relative Velocity

- When you are standing still, your velocity relative to the road is also 8 m/s, but your velocity relative to the bus is zero.

- A vector representation of this problem is shown in the figure.



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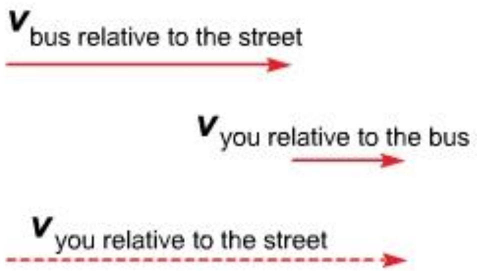
Relative Velocity

- When a coordinate system is moving, two velocities are added if both motions are in the same direction and one is subtracted from the other if the motions are in opposite directions.

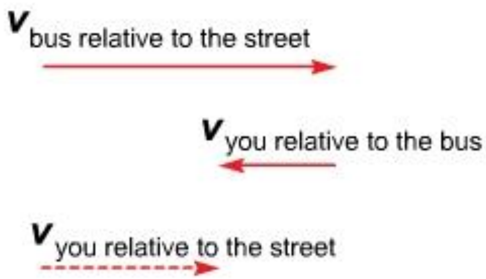
- In the given figure, you will find that your velocity relative to the street is 11 m/s, the sum of 8 m/s and 3 m/s.



Relative Velocity

- The adjoining figure shows that because the two velocities are in opposite directions, the resultant velocity is 5 m/s–the difference between 8 m/s and 3 m/s.

- You can see that when the velocities are along the same line, simple addition or subtraction can be used to determine the relative velocity.



Relative Velocity

- Mathematically, relative velocity is represented as $v_{y/b} + v_{b/r} = v_{y/r}$.
- The more general form of this equation is:

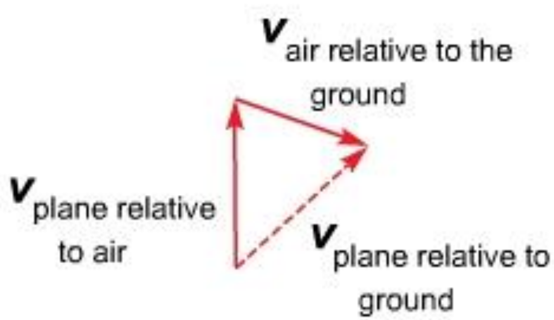
Relative Velocity $$v_{a/b} + v_{b/c} = v_{a/c}$$

- The relative velocity of object a to object c is the vector sum of object a's velocity relative to object b and object b's velocity relative to object c.

Relative Velocity

- The method for adding relative velocities also applies to motion in two dimensions.

- For example, airline pilots must take into account the plane's speed relative to the air, and their direction of flight relative to the air. They also must consider the velocity of the wind at the altitude they are flying relative to the ground.



Example

View Movie

Relative Velocity of a Marble

Ana and Sandra are riding on a ferry boat that is traveling east at a speed of 4.0 m/s. Sandra rolls a marble with a velocity of 0.75 m/s north, straight across the deck of the boat to Ana. What is the velocity of the marble relative to the water?

Relative Velocity of a Marble

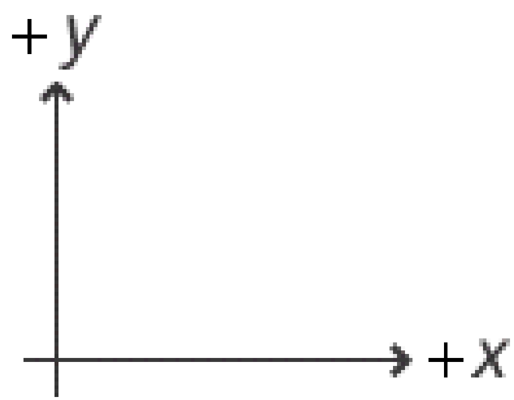
View Question

Step 1: Analyze and Sketch the Problem

Relative Velocity of a Marble

? View Question

Establish a coordinate system.



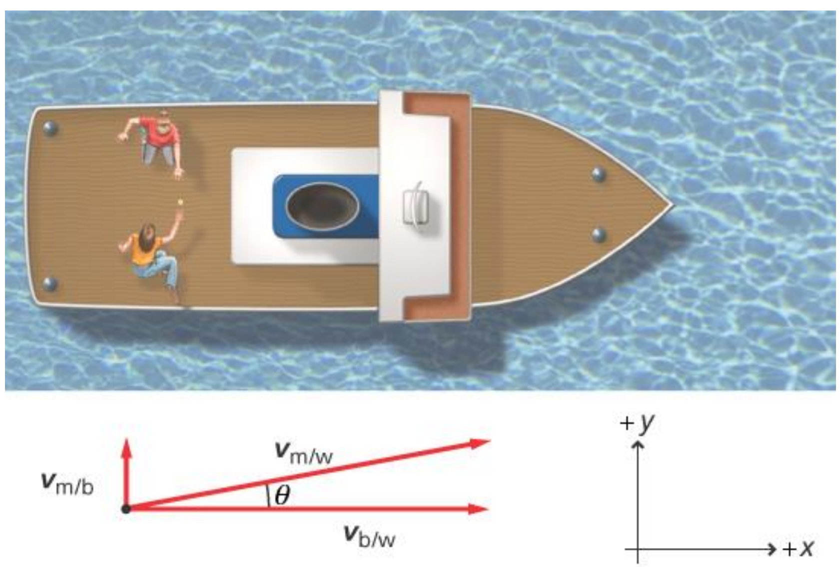
Section

6.3

Relative Velocity of a Marble

? View Question

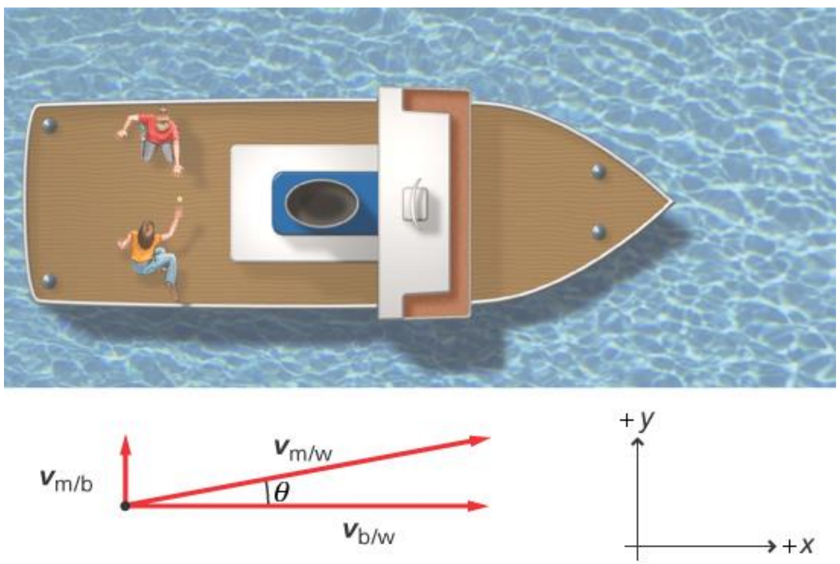
Draw vectors to represent the velocities of the boat relative to the water and the marble relative to the boat.



Relative Velocity of a Marble

View Question

Identify known and unknown variables.



Known:

$v_{b/w}$ = 4.0 m/s

$v_{m/b}$ = 0.75 m/s

Unknown:

$v_{m/w}$ = ?

Relative Velocity of a Marble

View Question

Step 2: Solve for the Unknown

Relative Velocity of a Marble

View Question

Because the two velocities are at right angles, use the Pythagorean theorem.

$$v_{m/w}^{2} = v_{b/w}^{2} + v_{m/b}^{2}$$

$$v_{m/w} = \sqrt{v_{b/w}^{2} + v_{m/b}^{2}}$$

Relative Velocity of a Marble

? View Question

Substitute $v_{b/w}$ = 4.0 m/s, $v_{m/b}$ = 0.75 m/s

$$v_{m/w} = \sqrt{(4.0\ \mathrm{m/s})^2 + (0.75\ \mathrm{m/s})^2}$$

$$= 4.1\ \mathrm{m/s}$$

Relative Velocity of a Marble

View Question

Find the angle of the marble's motion.

$$\theta = \tan^{-1}\left(\frac{v_{m/b}}{v_{b/w}}\right)$$

Relative Velocity of a Marble

View Question

Substitute $v_{b/w}$ = 4.0 m/s, $v_{m/b}$ = 0.75 m/s

$$\theta = \tan^{-1}\left(\frac{0.75\ \text{m/s}}{4.0\ \text{m/s}}\right)$$

= 11° north of east

The marble is traveling 4.1 m/s at 11° north of east.

Relative Velocity of a Marble

View Question

Step 3: Evaluate the Answer

Relative Velocity of a Marble

View Question

- **Are the units correct?**

  Dimensional analysis verifies that the velocity is in m/s.

- **Do the signs make sense?**

  The signs should all be positive.

- **Are the magnitudes realistic?**

  The resulting velocity is of the same order of magnitude as the velocities given in the problem.

Relative Velocity of a Marble

View Question

The steps covered were:

- **Step 1:** Analyze and Sketch the Problem
  - Establish a coordinate system.
  - Draw vectors to represent the velocities of the boat relative to the water and the marble relative to the boat.
- **Step 2:** Solve for the Unknown
  - Use the Pythagorean theorem.
- **Step 3:** Evaluate the Answer

Relative Velocity

- You can add relative velocities even if they are at arbitrary angles by using the graphical methods.
- The key to properly analyzing a two-dimensional relative-velocity situation is drawing the proper triangle to represent the three velocities. Once you have this triangle, you simply apply your knowledge of vector addition.
- If the situation contains two velocities that are perpendicular to each other, you can find the third by applying the Pythagorean theorem; however, if the situation has no right angles, you will need to use one or both of the laws of sines and cosines.

Question 1

Steven is walking on the roof of a bus with a velocity of 2 m/s toward the rear end of the bus. The bus is moving with a velocity of 10 m/s. What is the velocity of Steven with respect to Anudja sitting inside the bus and Mark standing on the street?

A. Velocity of Steven with respect to Anudja is 2 m/s and with respect to Mark is 12 m/s.

B. Velocity of Steven with respect to Anudja is 2 m/s and with respect to Mark is 8 m/s.

C. Velocity of Steven with respect to Anudja is 10 m/s and with respect to Mark is 12 m/s.

D. Velocity of Steven with respect to Anudja is 10 m/s and with respect to Mark is 8 m/s.

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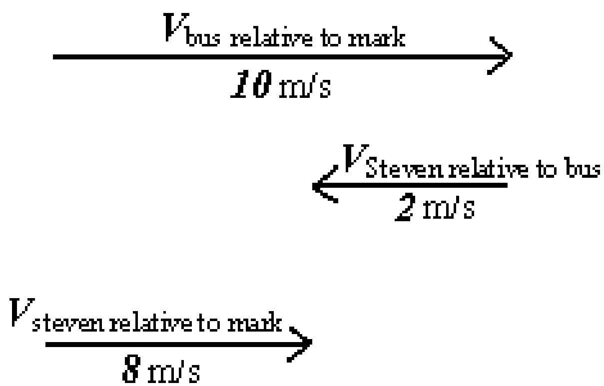
Answer 1

Answer: B

Reason: The velocity of Steven with respect to Anudja is 2 m/s since Steven is moving with a velocity of 2 m/s with respect to the bus, and Anudja is at rest with respect to the bus.

The velocity of Steven with respect to Mark can be understood with the help of the following vector representation.



Question 2

Which of the following formulas is the general form of relative velocity of objects a, b, and c?

A. $\mathbf{V}_{a/b} + \mathbf{V}_{a/c} = \mathbf{V}_{b/c}$

B. $\mathbf{V}_{a/b} - \mathbf{V}_{b/c} = \mathbf{V}_{a/c}$

C. $\mathbf{V}_{a/b} + \mathbf{V}_{b/c} = \mathbf{V}_{a/c}$

D. $\mathbf{V}_{a/b} - \mathbf{V}_{a/c} = \mathbf{V}_{b/c}$

Answer 2

Answer: C

Reason: Relative velocity law is $\boldsymbol{V}_{a/b} + \boldsymbol{V}_{b/c} = \boldsymbol{V}_{a/c}$.

The relative velocity of object a to object c is the vector sum of object a's velocity relative to object b and object b's velocity relative to object c.

Question 3

An airplane flies due south at 100 km/hr relative to the air. Wind is blowing at 20 km/hr to the west relative to the ground. What is the plane's speed with respect to the ground?

A. (100 + 20) km/hr

B. (100 − 20) km/hr

C. $\sqrt{100^2 + 20^2}$ km/hr

D. $\sqrt{100^2 - 20^2}$ km/hr

Answer 3

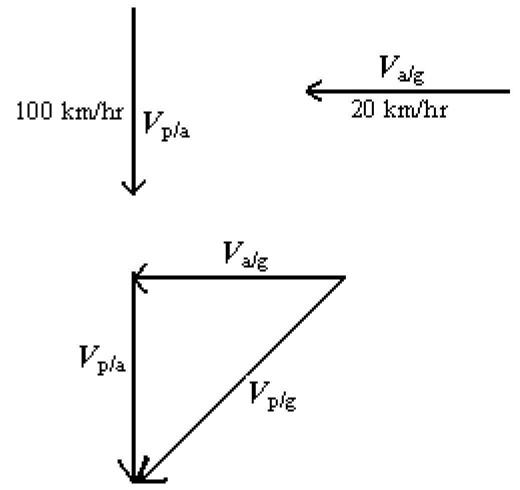
Answer: C

Reason: Since the two velocities are at right angles, we can apply Pythagoras theorem of addition law. By using relative velocity law, we can write:

$$V_{p/a}^{\ 2} + V_{a/g}^{\ 2} = V_{p/g}^{\ 2}$$

$$V_{p/g} = \sqrt{V_{p/a^2} + V_{a/g^2}}$$

$$= \sqrt{100^2 + 20^2}\ \text{km/hr}$$



End of Chapter

Centripetal Acceleration

- In the adjoining figure, you can determine the direction in which the hammer flies when the chain is released.
- Once the contact force of the chain is gone, there is no force accelerating the hammer toward the center of the circle.
- So the hammer lies off in the direction of its velocity, which is tangent to the circle.

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Relative Velocity

- Another example of combined relative velocities is the navigation of migrating neotropical songbirds.
- In addition to knowing in which direction to fly, a bird must account for its speed relative to the air and its direction relative to the ground.
- If a bird tries to fly over the Gulf of Mexico into too strong a headwind, it will run out of energy before it reaches the other shore and will perish.
- Similarly, the bird must account for crosswinds or it will not reach its destination.

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The Flight of a Ball

A ball is launched at 4.5 m/s at 66° above the horizontal. What are the maximum height and flight time of the ball?

Relative Velocity of a Marble

Ana and Sandra are riding on a ferry boat that is traveling east at a speed of 4.0 m/s. Sandra rolls a marble with a velocity of 0.75 m/s north, straight across the deck of the boat to Ana. What is the velocity of the marble relative to the water?