Lesson 4.5 Real-World Problems: Algebraic Inequalities

Solve.

Example

The average age of five people is at least 60 years old. The ages of four of the people are 50, 55, 63, and 60 years old. What is the minimum age of the fifth person?

Let x be the age of the fifth person.	Define the variable.
Average ≥ 60	
$\dfrac{50 + 55 + 63 + 60 + x}{5} \geq 60$	Write an inequality.
$\dfrac{228 + x}{5} \geq 60$	Simplify.
$5 \cdot \left(\dfrac{228 + x}{5}\right) \geq 5 \cdot 60$	Multiply both sides by 5.
$228 + x \geq 300$	Simplify.
$228 + x - 228 \geq 300 - 228$	Subtract 228 from both sides.
$x \geq 72$	Simplify.

The fifth person is at least _72 years old_ .

Complete.

1. The average of 30, 35, 35, 40, and a fifth number is at most 40. What is the maximum value of the fifth number?

   Let x be the fifth number.

Average ≤ 40	Define the variable.
$\dfrac{30 + \text{\_\_\_\_} + \text{\_\_\_\_} + \text{\_\_\_\_} + \text{\_\_\_\_}}{5} \leq 40$	Write an inequality.
$\dfrac{\text{_____}}{5} \leq 40$	Simplify.
$5 \cdot \left(\dfrac{\text{_____}}{5}\right) \leq 5 \cdot 40$	Multiply both sides by 5.
$140 + \text{\_\_\_\_} \leq \text{\_\_\_\_}$	Simplify.
$140 + \text{\_\_\_\_} - \text{\_\_\_\_} \leq \text{\_\_\_\_} - 140$	Subtract 140 from both sides.
$x \leq \text{\_\_\_\_}$	Simplify.

   The maximum value of the fifth number is at most _____.

Solve.

2. Jeremy bought four books. The costs of three books were $10, $13, and $15. The average cost of the four books is at least $14. What is the minimum cost of the fourth book?

3. The average score of six students on a mathematics test is at least 70. Five of the students scored 72, 65, 68, 70, and 78. What is the lowest score of the sixth student?

Solve.

> **Example**
>
> Kathie buys a bottle of fruit juice for $2.50 and a few loaves of bread for $1.20 each. If she has only $10, at most how many loaves of breads can she buy?
>
> *Let y be the number of loaves of breads Kathie can buy.*
>
> $$2.5 + 1.2x \leq 10$$ Write an inequality.
>
> $$2.5 + 1.2y - 2.5 \leq 10 - 2.5$$ Subtract 2.5 from both sides.
>
> $$1.2y \leq 7.5$$ Simplify.
>
> $$\frac{1.2y}{1.2} \leq \frac{7.5}{1.2}$$ Divide both sides by 1.2.
>
> $$y \leq 6.25$$ Simplify.
>
> Kathie can buy at most ____6____ loaves of bread.
>
> In this case, the greatest number of loaves of bread cannot be a decimal. It must be a whole number.

Complete.

4. Kevin's assignment is to complete a mathematics quiz with 25 multiple-choice questions. For every correct answer, he gets 2 points. One point is deducted for every wrong answer. How many answers must Kevin get correct in order to score more than 30 points?

Let x be the number of correct answers.
Then the number of wrong answers is $25 - x$.

So, the points awarded for correct answers is $2x$ and the scores deducted for wrong answers is $-1(25 - x)$.

$$2x + \underline{\hspace{1.5cm}} > 30 \qquad\qquad \text{Write an inequality.}$$

$$2x - \underline{\hspace{1cm}} + \underline{\hspace{1cm}} > 30$$

$$\underline{\hspace{1cm}} - \underline{\hspace{1cm}} > 30 \qquad\qquad \text{Simplify.}$$

$$3x - \underline{\hspace{1cm}} + \underline{\hspace{1cm}} > 30 + \underline{\hspace{1cm}} \qquad\qquad \text{Add} \underline{\hspace{1cm}} \text{to both sides.}$$

$$\underline{\hspace{1cm}} > \underline{\hspace{1cm}} \qquad\qquad \text{Simplify.}$$

$$\underline{\hspace{1cm}} > \underline{\hspace{1cm}} \qquad\qquad \text{Divide both sides by} \underline{\hspace{1cm}}.$$

$$x > \underline{\hspace{1cm}} \qquad\qquad \text{Simplify.}$$

Kevin must get _____ correct answers in order to score more than 30 points.

Solve.

5. Natalia has $20 to spend at a stationery shop. She plans to buy a notebook for $3.50 and some pens for $1.50 each. At most, how many pens can she buy?

6. There are 40 multiple-choice questions on a quiz. A student gets 3 points for every correct answer. One point is deducted for a wrong answer. If a student needs to score at least 100 points to pass the quiz, how many answers must he answer correctly?

Solve.

Example

Selena would like to sign up for a mobile phone plan and is given two different payment options.

Plan A	Plan B
$0.20 per minute $45 per month	$0.15 per minute $55 per month

After how many minutes of talk-time will Plan B be less expensive than Plan A?

Let x be the number of minutes of talk-time.

$0.2x + 45 > 0.15x + 55$	Write an inequality.
$0.2x + 45 - 0.15x > 0.15x + 55 - 0.15x$	Subtract $0.15x$ from both sides.
$0.05x + 45 > 55$	Simplify.
$0.05x + 45 - 45 > 55 - 45$	Subtract 45 from both sides.
$0.05x > 10$	Simplify.
$\dfrac{0.05x}{0.05} > \dfrac{10}{0.05}$	Divide both sides by 0.05.
$x > 200$	Simplify.

Plan B will be less expensive than Plan A if the talk-time exceeds *200* minutes.

Complete.

7. The membership fees for two different movie clubs are provided below.

Best Movie Club	Ultimate Movie Club
$50 administration fee $80 per month	$30 administration fee $100 per month

After how many months will Best Movie Club be less expensive than Ultimate Movie Club?

Let y be the number of months.

$50 + \text{\_\_\_\_\_} < 30 + \text{\_\_\_\_\_}$	Write an inequality.
$50 + \text{\_\_\_\_\_} - \text{\_\_\_\_\_} < 30 + \text{\_\_\_\_\_} - \text{\_\_\_\_\_}$	Subtract _____ from both sides.
$50 - \text{\_\_\_\_\_} < 30$	Simplify.
$50 - \text{\_\_\_\_\_} - \text{\_\_\_\_\_} < 30 - \text{\_\_\_\_\_}$	Subtract _____ from both sides.
$\text{\_\_\_\_\_} < \text{\_\_\_\_\_}$	Simplify.
$\text{\_\_\_\_\_} > \text{\_\_\_\_\_}$	Divide both sides by _____ and reverse the symbol.
$y > \text{\_\_\_\_\_}$	Simplify.

Best Movie Club will be less expensive than Ultimate Movie Club after _____ months.

Solve.

8. Felicia wants to bring some people on a day tour in New York City. She organized the packages from two travel agencies in the table shown.

Travel Agency A	Travel Agency B
$30 per person $65 agency fee	$50 per person $25 agency fee

What is the maximum number of people who can sign up for the trip so that Travel Agency B will be less expensive than Travel Agency A?

9. Emily would like to rent a studio for a dance audition. She was given the following packages to choose from.

Studio A	Studio B
$2,500 for six hours $10 per hour exceeded	$3,000 for six hours $5 per hour exceeded

How many hours would Emily have exceeded at Studio A to incur more cost than at Studio B?