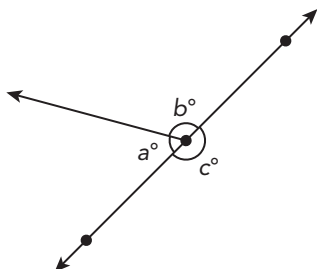


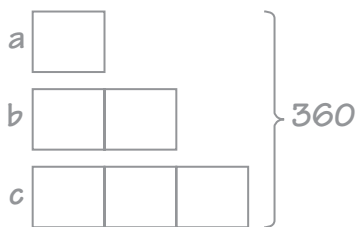
**Use ratios to find angle measures in a diagram.***Example*

In the diagram below, the ratio  $a : b : c = 1 : 2 : 3$ . Find the values of  $a$ ,  $b$ , and  $c$ .

**Method 1**

Draw a bar model.

$$a^\circ + b^\circ + c^\circ = 360^\circ$$



From the bar model,

$$6 \text{ units} \rightarrow 360$$

$$1 \text{ unit} \rightarrow \frac{360}{6} = 60$$

$$a = 60$$

$$b = 2 \cdot 60 = 120$$

$$c = 3 \cdot 60 = 180$$

**Method 2**

Use a variable to represent the measure of the angle.

The ratio  $a : b : c = 1 : 2 : 3$ . So,  $b = 2 \cdot a$  and  $c = 3 \cdot a$ .

$$a^\circ + b^\circ + c^\circ = 360^\circ$$

$$a + 2a + 3a = 360$$

$$6a = 360$$

$$\frac{6a}{6} = \frac{360}{6}$$

$$a = 60$$

$\angle$ s at a point

Substitute.

Simplify.

Divide both sides by 6.

Simplify.

$$b = 2 \cdot a$$

$$= 2 \cdot 60$$

$$= 120$$

$$c = 3 \cdot a$$

$$= 3 \cdot 60$$

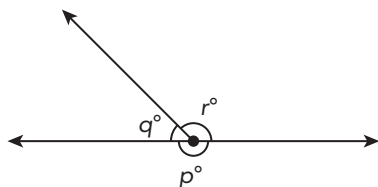
$$= 180$$

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**Complete.**

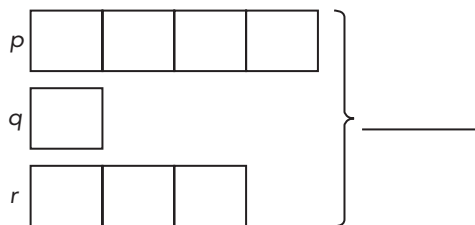
9. In the diagram below, the ratio  $p : q : r = 4 : 1 : 3$ . Find the values of  $p$ ,  $q$ , and  $r$ .



**Method 1**

Draw a bar model.

$$p^\circ + q^\circ + r^\circ = \underline{\hspace{2cm}}$$



From the bar model,

\_\_\_\_\_ units  $\rightarrow$  \_\_\_\_\_

1 unit  $\rightarrow$  \_\_\_\_\_ = \_\_\_\_\_

$$p = 4 \cdot \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$q = \underline{\hspace{1cm}}$$

$$r = 3 \cdot \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

**Method 2**

Use a variable to represent the measure of the angle.

The ratio  $p : q : r = 4 : 1 : 3$ . So,  $p = 4 \cdot q$  and  $r = 3 \cdot q$ .

$$p^\circ + q^\circ + r^\circ = \underline{\hspace{2cm}}$$

$\angle$ s at a point

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

Substitute.

$$\underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

Simplify.

$$\underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

Divide both sides by 8.

$$q = \underline{\hspace{1cm}}$$

Simplify.

$$p = 4 \cdot q$$

$$r = 3 \cdot q$$

$$= 4 \cdot \underline{\hspace{1cm}}$$

$$= 3 \cdot \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$

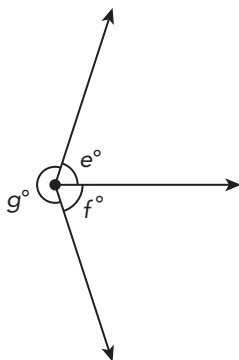
$$= \underline{\hspace{1cm}}$$

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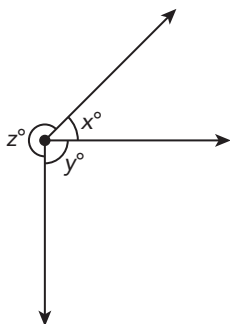
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**Find the value of each variable.**

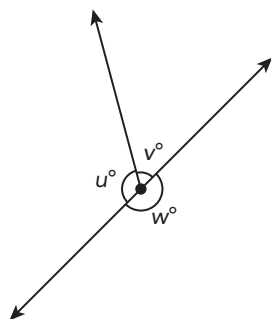
10. In the diagram below, the ratio  $e : f : g = 1 : 1 : 3$ .

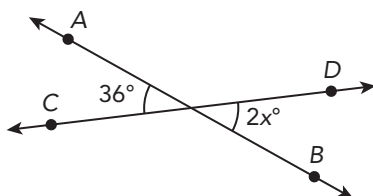


11. In the diagram below, the ratio  $x : y : z = 1 : 2 : 5$ .



12. In the diagram below, the ratio  $u : v : w = 2 : 1 : 3$ .



**Use algebra and vertical angles to find the value of each variable.***Example* $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$  are straight lines. Find the value of  $x$ .

$$2x^\circ = 36^\circ$$

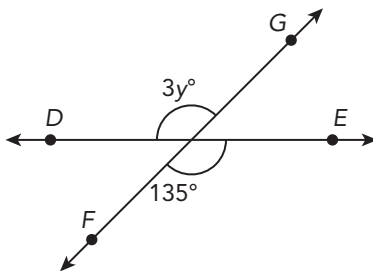
Vert.  $\angle$ s

$$\frac{2x}{2} = \frac{36}{2}$$

Divide both sides by 2.

$$x = 18$$

Simplify.

**Complete.**13.  $\overleftrightarrow{DE}$  and  $\overleftrightarrow{FG}$  are straight lines. Find the value of  $y$ .

$$3y^\circ = \underline{\hspace{2cm}}$$

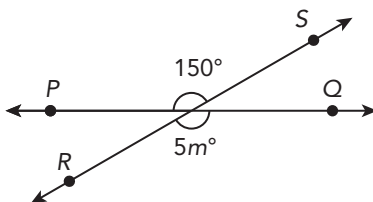
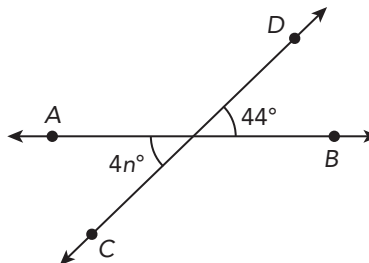
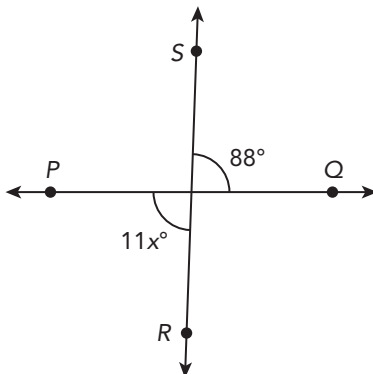
Vert.  $\angle$ s

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Divide both sides by \_\_\_\_\_.

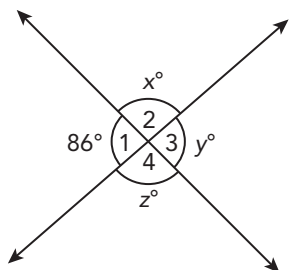
$$y = \underline{\hspace{2cm}}$$

Simplify.

**Use algebra and vertical angles to find the value of each variable.**14.  $\overleftrightarrow{PQ}$  and  $\overleftrightarrow{RS}$  are straight lines.15.  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$  are straight lines.16.  $\overleftrightarrow{PQ}$  and  $\overleftrightarrow{RS}$  are straight lines.

**Apply reasoning to find the value of each variable.***Example*

In the diagram, two straight lines intersect to form angles 1, 2, 3, and 4. Find the value of each variable if  $m\angle 1 = 86^\circ$ .



Make use of angle properties – supplementary angles on a straight line and vertical angles.

$$\begin{aligned} m\angle 1 + m\angle 2 &= 180^\circ \\ 86^\circ + x^\circ &= 180^\circ \\ 86^\circ + x^\circ - 86^\circ &= 180^\circ - 86^\circ \\ x &= 94^\circ \end{aligned}$$

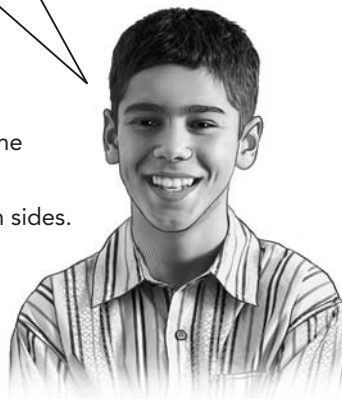
$$\begin{aligned} m\angle 3 &= m\angle 1 \\ y &= 86 \end{aligned}$$

$$\begin{aligned} m\angle 4 &= m\angle 2 \\ z &= 94 \end{aligned}$$

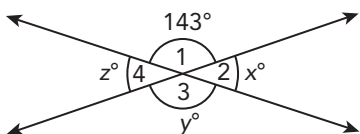
Adj.  $\angle$ s on a straight line  
Substitute.  
Subtract  $86^\circ$  from both sides.  
Simplify.

Vert.  $\angle$ s  
Substitute.

Vert.  $\angle$ s  
Substitute.

**Complete.**

17. In the diagram, two straight lines intersect to form angles 1, 2, 3, and 4. Find the value of each variable if  $m\angle 1 = 143^\circ$ .



$$m\angle 1 + m\angle 4 = 180^\circ$$

Adj.  $\angle$ s on a st. line

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 180^\circ$$

Substitute.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} - \underline{\hspace{2cm}} = 180^\circ - \underline{\hspace{2cm}}$$

Subtract  $143^\circ$  from both sides.

$$Z = \underline{\hspace{2cm}}$$

Simplify.

$$m\angle 3 = m\angle 1$$

$$m\angle 2 = m\angle 4$$

$$y = \underline{\hspace{2cm}}$$

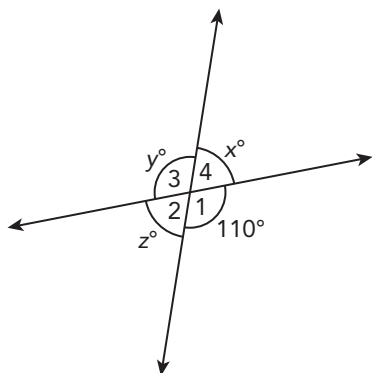
$$x = \underline{\hspace{2cm}}$$

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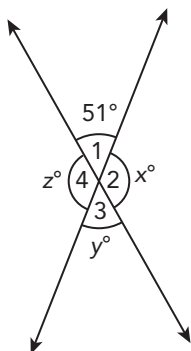
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**Apply reasoning to find the value of each variable.**

18. In the diagram, two straight lines intersect to form angles 1, 2, 3, and 4. Find the value of each variable if  $m\angle 1 = 110^\circ$ .



19. In the diagram, two straight lines intersect to form angles 1, 2, 3, and 4. Find the value of each variable if  $m\angle 1 = 51^\circ$ .



20. In the diagram, two straight lines intersect to form angle measures  $x^\circ$ ,  $y^\circ$ ,  $z^\circ$ , and  $159^\circ$ . Find the value of each variable.

